

# SAMPLE PAPER TEST 01 FOR BOARD EXAM (2023- 24)

SUBJECT: MATHEMATICS (041)

MAX. MARKS : 80

CLASS : XII

DURATION: 3 HRS

## General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

## SECTION – A

Questions 1 to 20 carry 1 mark each.

1. The matrix  $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$  is a symmetric matrix. Then the value of a and b respectively are:

- (a)  $\frac{-2}{3}, \frac{3}{2}$  (b)  $\frac{-1}{2}, \frac{1}{2}$  (c) -2, 2 (d)  $\frac{3}{2}, \frac{1}{2}$

2. If one root of the equation  $\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 7$  is  $x = -9$ , then the other two roots are:

- (a) 6, 3 (b) 6, -3 (c) -2, -7 (d) 2, 6

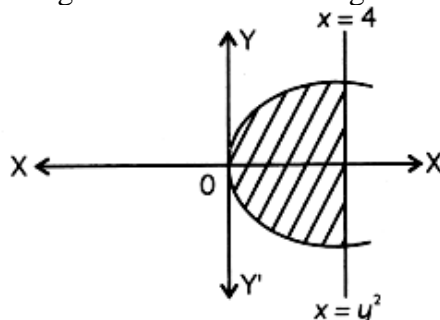
3. If  $f(x) = x \tan^{-1} x$ , then  $f'(1) =$

- (a)  $1 + \frac{\pi}{4}$  (b)  $\frac{1}{2} + \frac{\pi}{4}$  (c)  $\frac{1}{2} - \frac{\pi}{4}$  (d) 2

4. The value of  $\lambda$  such that the vector  $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$  are orthogonal is:

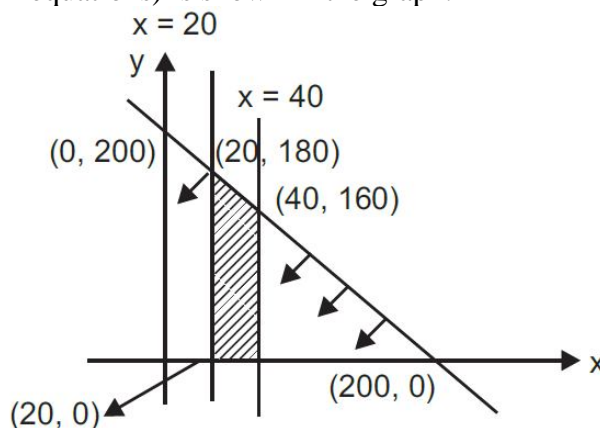
- (a)  $3/2$  (b)  $-5/2$  (c)  $-1/2$  (d)  $1/2$

5. The area (in sq. m) of the shaded region as shown in the figure is:



- (a)  $32/3$  sq. units (b)  $16/3$  sq. units (c) 4 sq. units (d) 16 sq. units

6. The order and the degree of the differential equation  $2x^2 \frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + y = 0$  are:  
 (a) 1, 1                      (b) 2, 1                      (c) 1, 2                      (d) 3, 1
7. A set of values of decision variables that satisfies the linear constraints and non-negativity conditions of an L.P.P. is called its:  
 (a) Unbounded solution                      (b) Optimum solution  
 (c) Feasible solution                      (d) None of these
8. The value of  $\int_0^a \frac{\sqrt{a}}{\sqrt{x} + \sqrt{a-x}} dx$  is:  
 (a)  $a/2$                       (b)  $a$                       (c)  $a^2$                       (d) 0
9. For any vector  $\vec{a}$ , the value of  $|\vec{a} \cdot \hat{i}|^2 + |\vec{a} \cdot \hat{j}|^2 + |\vec{a} \cdot \hat{k}|^2$  is:  
 (a)  $a$                       (b)  $a^2$                       (c) 1                      (d) 0
10. For an L.P.P. the objective function is  $Z = 400x + 300y$ , and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.



Find the coordinates at which the objective function is maximum.

- (a) (20, 0)                      (b) (40, 0)                      (c) (40, 160)                      (d) (20, 180)
11. A function  $f : Z \rightarrow Z$  given by  $f(x) = 5x + 3$  is  
 (a) one-one but not onto.                      (b) bijective  
 (c) onto but not one-one                      (d) None of these
12. The cofactor of (-1) in the matrix  $\begin{bmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{bmatrix}$  is:  
 (a) 1                      (b) 2                      (c) -1                      (d) 0
13. If A and B are two events such that  $P(A) = 1/2$ ,  $P(B) = 1/3$  and  $P(A/B) = 1/4$ , then  $P(A' \cap B')$  equals  
 (a)  $1/12$                       (b)  $3/4$                       (c)  $1/4$                       (d)  $3/16$
14. Let A be a non-singular matrix of order  $(3 \times 3)$ . Then  $|\text{adj.}A|$  is equal to  
 (a)  $|A|$                       (b)  $|A|^2$                       (c)  $|A|^3$                       (d)  $3|A|$
15. The general solution of the differential equation  $\frac{dy}{dx} = 2^{-y}$  is:  
 (a)  $2y = x \log 2 + C \log 2$                       (b)  $2y = x \log 3 - C \log 3$   
 (c)  $y = x \log 2 - C \log 2$                       (d) None of these

16. The domain, for which  $\tan^{-1}x > \cot^{-1}x$  holds true, is:  
 (a)  $x = 1$  (b)  $x > 1$  (c)  $x < 1$  (d) Not defined
17. A point that lies on the line  $\frac{x-1}{-2} = \frac{y+3}{4} = \frac{1-z}{7}$  is:  
 (a) (1, -3, 1) (b) (-2, 4, 7) (c) (-1, 3, 1) (d) (2, -4, -7)
18. The direction ratios of the line  $6x - 2 = 3y + 1 = 2z - 2$  are:  
 (a) 6, 3, 2 (b) 1, 1, 2 (c) 1, 2, 3 (d) 1, 3, 2

### ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.  
 (b) Both A and R are true but R is not the correct explanation of A.  
 (c) A is true but R is false.  
 (d) A is false but R is true.

19. **Assertion (A):** Lines  $\frac{x+2}{-2} = \frac{y-1}{3} = \frac{z-z}{1}$  and  $\frac{x-3}{-3} = \frac{y}{-2} = \frac{z+1}{2}$  are coplanar.

**Reason (R):** Let line  $l_1$  passes through the point  $(x_1, y_1, z_1)$  and parallel to the vector whose direction ratios are  $a_1, b_1$  and  $c_1$ ; and let line  $l_2$  passes through the point  $(x_2, y_2, z_2)$  and parallel to the vector whose direction ratios are  $a_2, b_2$  and  $c_2$ .

Then both lines  $l_1$  and  $l_2$  are coplanar if and only if 
$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

20. **Assertion (A):**  $\sin^{-1}(\sin(2\pi/3)) = 2\pi/3$

**Reason (R):**  $\sin^{-1}(\sin \theta) = \theta$ , if  $\theta \in [(-\pi)/2, \pi/2]$

### SECTION – B

Questions 21 to 25 carry 2 marks each.

21. Check whether the relation R in the set Z of integers defined as  $R = \{(a, b) : a + b \text{ is "divisible by 2"}\}$  is reflexive, symmetric or transitive. Write the equivalence class containing 0 i.e.,  $[0]$ .

OR

Express  $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$ ,  $-\frac{3\pi}{2} < x < \frac{\pi}{2}$  in simplest form.

22. Show that the function  $f(x) = x^3 - 3x^2 + 6x - 100$  is increasing on R.

23. Discuss the continuity of the following function at  $x = 0$  :

$$f(x) = \begin{cases} \frac{x^4 + 2x^3 + x^2}{\tan^{-1} x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

24. Find the vector equation of the line joining (1, 2, 3) and (-3, 4, 3) and show that it is perpendicular to the z-axis.

25. Prove that the points A, B and C with position vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  respectively are collinear if and only if  $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

OR

If  $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$  and  $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ , then find the projection of  $\vec{b}$  on  $\vec{a}$ .

### **SECTION – C**

Questions 13 to 22 carry 3 marks each.

26. Find the value of  $\int_2^4 \frac{x^2 + x}{\sqrt{2x+1}} dx$ .

OR

Find the value of  $\int_1^2 \frac{dx}{x(1 + \log x)^2}$ .

27. Evaluate:  $\int \frac{3x+1}{(x-1)^2(x+3)} dx$

28. Solve the following differential equation:  $\frac{dy}{dx} = x^3 \sec y$ , given that  $y(0) = 0$ .

OR

Solve the following differential equation:  $\left[ x \sin^2 \left( \frac{y}{x} \right) - y \right] dx + x dy = 0$

29. Solve the following Linear Programming Problem graphically:

Maximise  $z = 8x + 9y$  subject to the constraints:  $2x + 3y \leq 6$ ,  $3x - 2y \leq 6$ ,  $y \leq 1$ ;  $x, y \geq 0$

30. Two numbers are selected at random (without replacement) from first 7 natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X.

OR

There are three coins, one is a two headed coin (having head on both the faces), another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows head. What is probability that it was the two headed coin?

31. Evaluate:  $\int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}}$

### **SECTION – D**

Questions 32 to 35 carry 5 marks each.

32. Using integration, find the smaller area enclosed by the circle  $x^2 + y^2 = 4$  and the line  $x + y = 2$ .

33. Find the vector equation of the line through the point  $(1, 2, -4)$  and perpendicular to the two lines  $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$  and  $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

OR

Find the shortest distance between the following lines :

$$l_1 : \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$l_2 : \vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

**34.** Define the relation R in the set  $N \times N$  as follows:

For  $(a, b), (c, d) \in N \times N$ ,  $(a, b) R (c, d)$  iff  $ad = bc$ . Prove that R is an equivalence relation in  $N \times N$ .

**OR**

Given a non-empty set X, define the relation R in  $P(X)$  as follows:

For  $A, B \in P(X)$ ,  $(A, B) \in R$  iff  $A \subset B$ . Prove that R is reflexive, transitive and not symmetric.

**35.** If  $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$ , then find  $A^{-1}$  and use it to solve the following system of the equations :

$$x + 2y - 3z = 6, 3x + 2y - 2z = 3, 2x - y + z = 2$$

### **SECTION – E(Case Study Based Questions)**

**Questions 35 to 37 carry 4 marks each.**

**36. Case-Study 1:** Read the following passage and answer the questions given below.



There are two anti-aircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

- What is the probability that the shell fired from exactly one of them hit the plane?
- If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

**37. Case-Study 2:** Read the following passage and answer the questions given below.



The temperature of a person during an intestinal illness is given by

$$f(x) = -\frac{1}{10}x^2 + mx + \frac{493}{5}, 0 \leq x \leq 12, m \text{ being a constant, where } f(x) \text{ is the temperature in } ^\circ\text{F at } x \text{ days.}$$

- (i) Is the function differentiable in the interval  $(0, 12)$ ? Justify your answer.  
(ii) If 6 is the critical point of the function, then find the value of the constant

(iii) Find the intervals in which the function is strictly increasing/strictly decreasing.

**OR**

(iii) Find the points of local maximum/local minimum, if any, in the interval  $(0, 12)$  as well as the points of absolute maximum/absolute minimum in the interval  $[0, 12]$ . Also, find the corresponding local maximum/local minimum and the absolute maximum/absolute minimum values of the function.

### 38. Case-Study 3:

Sonam wants to prepare a sweet box for Diwali at home. For making lower part of box, she takes a square piece of cardboard of side 18 cm.

Now,  $x$  cm be the length of each side of the square cardboard which is to be cut off from corner of the square piece of side 18 cm.



Based on the above information, answer the following questions :

(i) Express Volume of the open box formed by folding up the cutting corner in terms of  $x$  and

find the value of  $x$  for which  $\frac{dV}{dx} = 0$ .

(ii) Sonam is interested in maximising the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?

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## SAMPLE PAPER TEST 2 FOR BOARD EXAM (2023- 24)

SUBJECT: MATHEMATICS (041)

MAX. MARKS : 80

CLASS : XII

DURATION: 3 HRS

### General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

### SECTION – A

Questions 1 to 20 carry 1 mark each.

1. The value of the expression  $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2$  is  
(a)  $\vec{a} \cdot \vec{b}$  (b)  $|\vec{a}| \cdot |\vec{b}|$  (c)  $|\vec{a}|^2 |\vec{b}|^2$  (d)  $(\vec{a} \cdot \vec{b})$
2. For what value of k the function  $f(x) = \begin{cases} 2x+1; & \text{if } x < 2 \\ k, & x = 2 \\ 3x-1; & x > 2 \end{cases}$  is continuous at  $x = 2$ ,  
(a) Any real value (b) No real value (c) 5 (d) 1/5
3. If  $A = \begin{bmatrix} -a & b \\ c & a \end{bmatrix}$  and  $A^2 = I$ , then  
(a)  $a^2 + bc - 1 = 0$  (b)  $1 - a^2 + bc = 0$  (c)  $a^2 + bc + 1 = 0$  (d)  $a^2 - bc + 1 = 0$
4. The Integrating factor of the differential equation  $(1 - y^2) \frac{dx}{dy} + yx = ay$  is  
(a)  $\frac{1}{y^2-1}$  (b)  $\frac{1}{\sqrt{y^2-1}}$  (c)  $\frac{1}{1-y^2}$  (d)  $\frac{1}{\sqrt{1-y^2}}$
5. If A is a square matrix of order 3 such that  $|A| = -5$ , then value of  $|-AA'|$  is  
(a) 125 (b) -125 (c) 25 (d) -25
6. If  $f'(x) = x^2 e^{x^3}$ , then  $f(x)$  is  
(a)  $\frac{1}{3} e^{x^3} + C$  (b)  $\frac{1}{3} e^{x^4} + C$  (c)  $\frac{1}{2} e^{x^3} + C$  (d)  $\frac{1}{2} e^{x^2} + C$
7. The sum of the order and the degree of the differential equation  $\frac{d}{dx} \left[ \left( \frac{dy}{dx} \right)^4 \right] = 0$  is  
(a) 1 (b) 2 (c) 3 (d) 4
8. The value of  $(\hat{i} \times \hat{j}) \cdot \hat{k} + (\hat{j} \times \hat{k}) \cdot \hat{i} + (\hat{k} \times \hat{i}) \cdot \hat{j}$  is  
(a) 0 (b) -1 (c) 1 (d) 3

9. Corner points of the feasible region for an LPP are (0, 3), (1,1) and (3,0). Let  $Z = px + qy$ , where  $p, q > 0$ , be the objective function. The condition on  $p$  and  $q$  so that the minimum of  $Z$  occurs at (3,0) and (1,1) is
- (a)  $p = q$  (b)  $p = \frac{q}{2}$  (c)  $p = 3q$  (d)  $p = q$
10.  $\int \frac{dx}{\sqrt{9x-4x^2}}$  equals
- (a)  $\frac{1}{9} \sin^{-1} \left( \frac{9x-8}{8} \right) + C$  (b)  $\frac{1}{2} \sin^{-1} \left( \frac{8x-9}{9} \right) + C$   
(c)  $\frac{1}{3} \sin^{-1} \left( \frac{9x-8}{8} \right) + C$  (d)  $\frac{1}{2} \sin^{-1} \left( \frac{9x-8}{8} \right) + C$
11. If  $A$  is a  $3 \times 3$  matrix and  $|A| = -2$  then value of  $|A(\text{adj}A)|$  is
- (a) -2 (b) 2 (c) -8 (d) 8
12. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let  $F = 4x + 6y$  be the objective function. The Minimum value of  $F$  occurs at
- (a) (0, 2) only (b) (3, 0) only  
(c) the mid point of the line segment joining the points (0, 2) and (3, 0) only  
(d) any point on the line segment joining the points (0, 2) and (3, 0).
13. If  $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ , then  $x$  is equal to
- (a) 6 (b)  $\pm 6$  (c) -6 (d) 0
14. If  $A$  is a square matrix of order 3, such that  $A(\text{adj}A) = 10I$ , then  $|\text{adj} A|$  is equal to
- (a) 1 (b) 10 (c) 100 (d) 101
15. Given two independent events  $A$  and  $B$  such that  $P(A) = 0.3$ ,  $P(B) = 0.6$  and  $P(A' \cap B)$  is
- (a) 0.42 (b) 0.18 (c) 0.28 (d) 0.12
16. If  $y = 5e^{7x} + 6e^{-7x}$ , show that  $\frac{d^2y}{dx^2}$  is equal to
- (a)  $7y$  (b)  $6y$  (c)  $49y$  (d)  $36y$
17. The projection of  $\vec{a}$  on  $\vec{b}$ , if  $|\vec{a}| = 8$  and  $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ .
- (a)  $\frac{8}{7}$  (b)  $\frac{2}{3}$  (c)  $\frac{2}{9}$  (d)  $\frac{4}{5}$
18. If the direction cosines of a line are  $k, k, k$  then
- (a)  $k > 0$  (b)  $0 < k < 1$  (c)  $k = 1$  (d)  $k = \frac{1}{\sqrt{3}}$  or  $k = -\frac{1}{\sqrt{3}}$

### ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.  
(b) Both A and R are true but R is not the correct explanation of A.  
(c) A is true but R is false.  
(d) A is false but R is true.



**19. Assertion(A) :** The pair of lines given by  $\vec{r} = \hat{i} - \hat{j} + \lambda(2\hat{i} + \hat{k})$  and  $\vec{r} = 2\hat{i} - \hat{k} + \mu(\hat{i} + \hat{j} - \hat{k})$  intersect .

**Reason(R) :** Two lines intersect each other, if they are not parallel and shortest distance = 0.

**20. Assertion (A) :** The value of expression  $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) + \tan^{-1}1 + \sin^{-1}\left(\frac{1}{2}\right)$  is  $\frac{\pi}{4}$

**Reason (R) :** Principal value branch of  $\sin^{-1}x$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  and that of  $\sec^{-1}x$  is  $[0, \pi] - \{\pi/2\}$

## **SECTION – B**

**Questions 21 to 25 carry 2 marks each.**

**21.** Write the simplest form of  $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}, x \neq 0$

**OR**

Show that  $f : \mathbb{N} \rightarrow \mathbb{N}$ , given by  $f(n) = \begin{cases} n+1, & \text{if } n \text{ is odd} \\ n-1, & \text{if } n \text{ is even} \end{cases}$  is a bijection.

**22.** An edge of a variable cube is increasing at the rate of 5cm per second. How fast is the volume increasing when the side is 15 cm.

**23.** If  $\vec{a} = 5\hat{i} - \hat{j} + 7\hat{k}$  and  $\vec{b} = \hat{i} - \hat{j} + \lambda\hat{k}$ , then find the value of  $\lambda$  so that the vectors  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  are orthogonal.

**OR**

Find the direction ratio and direction cosines of a line parallel to the line whose equations are  $6x - 2 = 3y + 1 = 2z - 4$

**24.** If  $x \sin(a+y) + \sin a \cos(a+y) = 0$ , then prove that  $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

**25.** Find  $|\vec{x}|$  if  $(\vec{x} - \vec{a})(\vec{x} + \vec{a}) = 12$ , where  $\vec{a}$  is a unit vector.

## **SECTION – C**

**Questions 26 to 31 carry 3 marks each.**

**26.** Evaluate:  $\int \frac{1}{9x^2 + 6x + 5} dx$

**27.** Probabilities of solving specific problem independently by A and B are  $\frac{1}{2}$  and  $\frac{1}{3}$  respectively. If both try to solve the problem independently. Find the probability that (i) the problem is solved (ii) exactly one of them solves the problems.

**OR**

From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

**28.** Evaluate :  $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$

**OR**

Evaluate:  $\int_{-5}^5 |x + 2| dx$

**29.** Solve the differential equation:  $(1 - y^2)(1 + \log x) dx + 2xy dy = 0$

**OR**

Solve the differential equation  $x dy - y dx = \sqrt{x^2 + y^2} dx$

30. Minimize and maximize  $Z = 600x + 400y$   
Subject to the constraints:  $x + 2y \leq 12$ ;  $2x + y \leq 12$ ;  $4x + 5y \leq 20$ ;  $x \geq 0$ ;  $y \geq 0$  by graphical method
31. Evaluate:  $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$

### SECTION – D

Questions 32 to 35 carry 5 marks each.

32. Find the area of the region in the first quadrant enclosed by the x-axis, the line  $y = x$  and the circle  $x^2 + y^2 = 32$ .
33. An insect is crawling along the line  $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$  and another insect is crawling along the line  $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ . At what points on the lines should they reach so that the distance between them is the shortest? Find the shortest possible distance between them.

**OR**

The equation of motion of a rocket are:  $x = 2t$ ,  $y = -4t$ ,  $z = 4t$ , where the time  $t$  is given in seconds, and the coordinates of a moving point in km. What is the path of the rocket? At what distances will the rocket be from the starting point  $O(0,0,0)$  and from the following line in 10 seconds?

$$\vec{r} = 20\hat{i} - 10\hat{j} + 40\hat{k} + \mu(10\hat{i} - 20\hat{j} + 10\hat{k})$$

34. Show that the relation  $R$  on the set  $Z$  of all integers, given by  $R = \{(a, b) : 2 \text{ divides } (a - b)\}$  is an equivalence relation.

**OR**

Let  $A = R - \{3\}$  and  $B = R - \{1\}$ . Prove that the function  $f: A \rightarrow B$  defined by  $f(x) = \left( \frac{x-2}{x-3} \right)$  is  $f$  one-one and onto ? Justify your answer.

35. Given  $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ , verify that  $BA = 6I$ , how can we use the result

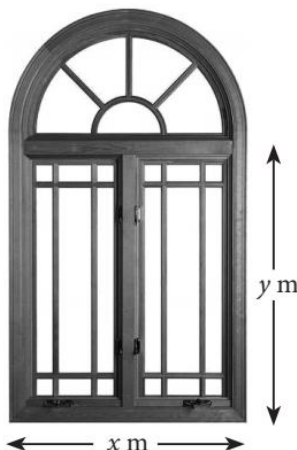
to find the values of  $x, y, z$  from given equations  $x - y = 3$ ,  $2x + 3y + 4z = 17$ ,  $y + 2z = 17$

### SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. **Case-Study 1:** Read the following passage and answer the questions given below.

Rohan, a student of class XII, visited his uncle's flat with his father. He observe that the window of the house is in the form of a rectangle surmounted by a semicircular opening having perimeter 10 m as shown in the figure.



- (i) If  $x$  and  $y$  represents the length and breadth of the rectangular region, then find the relation between  $x$  and  $y$ . Also, Find the expression of Area ( $A$ ) of the window. [2]
- (ii) Find the value of  $x$  for maximizing the Area ( $A$ ) of whole window. [2]
- (iii) Find the maximum area of the window.

**OR**

- (iii) For maximum value of  $A$ , find the breadth of the rectangular part of the window. [2]

### 37. Case-Study 2:

One day Shweta's Mathematics teacher was explaining the topic Increasing and decreasing functions in the class. He explained about different terms like stationary points, turning points etc. He also explained about the conditions for which a function will be increasing or decreasing. He took examples of different functions to make it more clear to the students. He then took the function  $f(x) = (x + 1)^3(x - 3)^3$  and ask the students to answer the following questions. With Shweta, you can also test your knowledge by answering the questions



- (i) Find the stationary points on the curve. [2]
- (ii) Find the intervals where the function is increasing and decreasing? [2]

### 38. Case-Study 3:

Mahindra Tractors is India's leading farm equipment manufacturer. It is the largest tractor selling factory in the world. This factory has two machine A and B. Past record shows that machine A produced 60% and machine B produced 40% of the output(tractors). Further 2% of the tractors produced by machine A and 1% produced by machine B were defective. All the tractors are put into one big store hall and one tractor is chosen at random.



- (i) Find the total probability of chosen tractor (at random) is defective.
- (ii) If in random choosing, chosen tractor is defective ,then find the probability that the chosen tractor is produced by machine 'B'

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